

29th Oct

Unit 2 Progress Check: FRQ Part A

- 1.
- 
- A GRAPHING CALCULATOR IS REQUIRED FOR THIS QUESTION.

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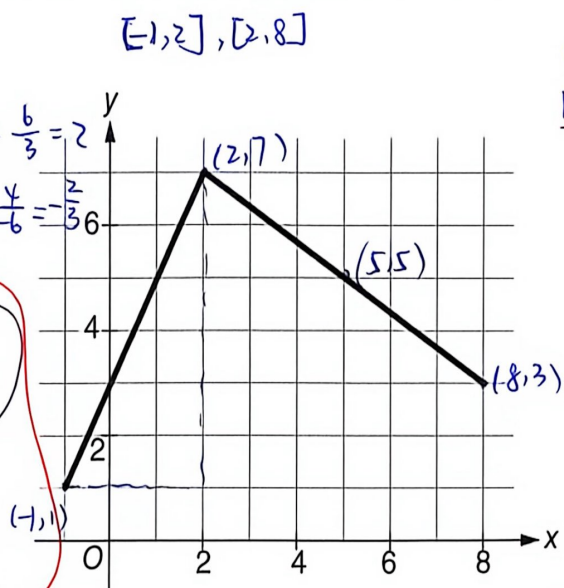
Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

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from the graph.

$$\text{when } x \in [-1, 2] \quad f'(x) = \frac{7-1}{2-(-1)} = \frac{6}{3} = 2$$

$$\text{when } x \in [2, 8] \quad f'(x) = \frac{7-3}{2-8} = \frac{4}{-6} = -\frac{2}{3}$$

Remember
the Chain RuleProduct &
Quotient RuleGraph of f

→ piecewise function.

Let f be the continuous function defined on $[-1, 8]$ whose graph, consisting of two line segments, is shown above. Let g and h be the functions defined by $g(x) = \sqrt{x^2 - x + 3}$ and $h(x) = 5e^x - 9 \sin x$.

- (a) The function
- k
- is defined by
- $k(x) = f(x)g(x)$
- . Find
- $k'(0)$
- .

- (b) The function
- m
- is defined by
- $m(x) = \frac{f(x)}{2g(x)}$
- . Find
- $m'(5)$
- .

- (c) Find the value of
- x
- for
- $-1 < x < 2$
- such that
- $f'(x) = h'(x)$
- .

$$\text{1) } h'(x) = 5e^x - 9 \cos x$$

$$f'(x) = 2 \quad (\text{because } -1 < x < 2)$$

$$5e^x - 9 \cos x = 2$$

$$x \approx 0.622$$

学会用 ☐

在FRQ中 highlight answer.

$$\text{1) } k(x) = f(x)g(x) \quad \text{Product Rule}$$

$$k'(x) = f'(x)g(x) + g'(x)f(x)$$

$$g'(x) = \left((x^2 - x + 3)^{\frac{1}{2}} \right)'$$

$$= \frac{1}{2} (x^2 - x + 3)^{-\frac{1}{2}} \cdot (2x - 1)$$

Chain Rule

$$= \frac{2x-1}{2\sqrt{x^2-x+3}}$$

$$x=0 \quad g'(x) = \frac{-1}{2\sqrt{3}}$$

$$f(0) = 3$$

$$g(0) = \sqrt{3}$$

$$f'(0) = 2$$

$$\therefore k'(0) = 2\sqrt{3} + 3 \times \frac{-1}{2\sqrt{3}}$$

$$= 2\sqrt{3} - \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{2}$$

Quotient Rule

$$\text{2) } m(x) = \frac{f(x)}{2g(x)}$$

$$\Rightarrow m'(x) = \frac{f'(x) \cdot 2g(x) - 2f(x)g'(x)}{(2g(x))^2}$$

$$g(5) = \sqrt{23}, \quad g'(5) = \frac{9}{2\sqrt{23}}$$

$$f(5) = 5, \quad f'(5) = -\frac{2}{3}$$

$$m'(5) = \frac{-\frac{2}{3}\sqrt{23} - 5(\frac{9}{2\sqrt{23}})}{2\sqrt{23}} \approx -0.171$$

ask for

Page 1 of 1

(3 places)

Unit 2 Progress Check: FRQ Part B

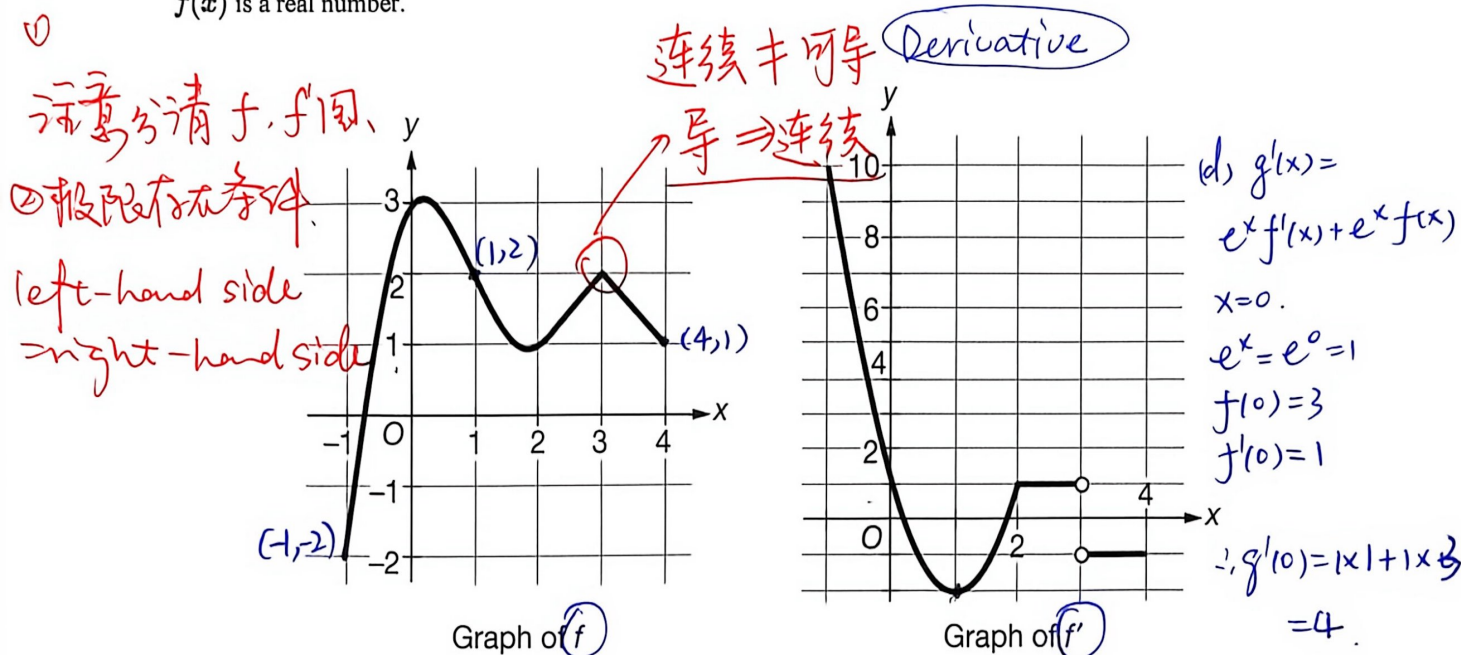
29th Oct.

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The graphs of the function f and its derivative f' are shown above for $-1 \leq x \leq 4$.

(a) Find the average rate of change of f over the interval $-1 \leq x \leq 4$. For how many values of x in the interval $-1 \leq x \leq 4$ does the instantaneous rate of change of f equal the average rate of change of f over that interval?

(b) Write an equation for the line tangent to the graph of f at $x = 1$.

(c) For each of $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}$ and $\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$, find the value or give a reason why it does not exist.

(d) Let g be the function defined by $g(x) = e^x f(x)$. Find $g'(0)$.

① (a) Average Rate of Change

$$= \frac{f(4) - f(-1)}{4 - (-1)} = \frac{1 - (-2)}{5} = \frac{3}{5}$$

② Two points.

(b) when $x = 1$ $f'(x) = -2$.

$$y - 2 = -2(x - 1)$$

$$y = -2x + 4$$

(c) $\lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2}$ means $f'(2)$.

from the second graph. $f'(2) = 1$.

$$\therefore \lim_{x \rightarrow 2} \frac{f(x) - f(2)}{x - 2} = 1.$$

$\lim_{x \rightarrow 3} \frac{f(x) - f(3)}{x - 3}$ means $f'(3)$

the same as $f'(2)$. $f'(3) \text{ DNE}$

because left-hand slope = 1, the right-hand slope = -1

Unit 2 Progress Check: FRQ Part B

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t (hours)	0	1	3	6	8
$P(t)$ (passengers)	0	35	204	600	728

The number of passengers who have boarded a ship is modeled by the differentiable function P , where t is the number of hours since boarding began. Values of $P(t)$ for selected values of t are given in the table above.

(a) According to the model, what is the average rate at which passengers board the ship, in passengers per hour, over the time interval $1 \leq t \leq 8$ hours?

(b) Write $P'(4.5)$ as the limit of a difference quotient. Use the data in the table to approximate $P'(4.5)$. Show the computations that lead to your answer.

(c) Must there be a time t , for $3 \leq t \leq 6$, at which $P(t) = 500$? Justify your answer.

(d) The total number of gallons of water used by the passengers on the ship is modeled by the function G defined by $G(t) = 120t\sqrt{t}$ for $0 \leq t \leq 8$, where t is the number of hours since boarding began. Find $G'(4)$, the rate at which passengers use water, in gallons per hour, at time $t = 4$ hours.

(a) AR of C = $\frac{P(8) - P(1)}{8 - 1} = \frac{728 - 35}{8 - 1} = \boxed{99}$ passengers per hour

(b) $P'(4.5) = \lim_{h \rightarrow 0} \frac{P(4.5 + h) - P(4.5)}{h}$

$P'(4.5) \approx \frac{P(6) - P(3)}{6 - 3} = \frac{600 - 204}{3} = \boxed{132}$ passengers per hour

15 清楚 meanings

(d) $G(t) = 120t\sqrt{t}$
 $= 120t^{\frac{3}{2}}$
 $G'(t) = 120 \cdot \frac{3}{2} \cdot t^{\frac{1}{2}}$
 $= 180\sqrt{t}$

$G'(4) = 180\sqrt{4} = 360$
gallons per hour.

介值定理:
可导性连续

Since P is differentiable and continuous on $[3, 6]$.

From the table $P(3) = 204 < 500 < 600 = P(6)$.

By IVT $\exists t \in (3, 6)$ satisfy $P(t) = 500$. So, yes.

罗列清楚
每一个条件

Unit 3 Progress Check: FRQ Part A

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x	-3	0	3	6
$f(x)$	-5	4	1	7
$f'(x)$	-1	2	-2	4

所有求导公式 $f(x)$ 只有里面 single x .
才不用 chain Rule.
其他都要 chain Rule.
(-x) 都要.

The table above gives values of a twice-differentiable function f and its first derivative f' for selected values of x . Let g be the function defined by $g(x) = f(x^2 - x)$.

- (a) What is the value of $g'(3)$?

- (b) It is known that $g''(0) = -1$. What is the value of $f''(0)$?

- (c) Is there a value c , for $0 < c < 3$, such that $g(c) = 5$? Justify your answer.

- (d) Let h be the function with derivative given by $h'(x) = 4e^{\cos x}$. At what value of x in the interval $-3 \leq x \leq 0$ does the instantaneous rate of change of h equal the average rate of change of f over the interval $-3 \leq x \leq 0$?

$$(a) g'(x) = f'(x^2 - x)(2x - 1)$$

$$g'(3) = f'(6)(5)$$

$$= 4 \times 5 = 20$$

Product Rule
but $f'(x^2 - x)$ [Chain Rule]

$$(b) g''(x) = f''(x^2 - x)(2x - 1)(2x - 1) + f'(x^2 - x)(2)$$

$$g''(0) = f''(0)(-1)(-1) + f'(0) \times 2 = -1$$

$$f''(0) + 4 = -1$$

$$f''(0) = -5$$

(c) Since f is differentiable and continuous, so $g(x) = f(x^2 - x)$ is continuous.

$$\text{At the endpoints, } g(0) = f(0) = 4$$

$$g(3) = f(6) = 7$$

Since $4 < 5 < 7$, by IVT, guarantees some $c \in (0, 3)$ such that $g(c) = 5$. $\boxed{\text{YES}}$

$$(d) \text{ Avg } C = \frac{f(0) - f(-3)}{0 - (-3)}$$

$$= \frac{4 - (-5)}{3} = 3$$

$$h'(x) = 4e^{\cos x} = 3$$

$$e^{\cos x} = \frac{3}{4}$$

$$\cos x = \ln\left(\frac{3}{4}\right)$$

$$x \approx -1.862$$

Unit 3 Progress Check: FRQ Part B

28th Oct

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明显是 Implicit

Consider the curve given by the equation $(2y+1)^3 - 24x = -3$. $\rightarrow$

(a) Show that $\frac{dy}{dx} = \frac{4}{(2y+1)^2}$.

(b) Write an equation for the line tangent to the curve at the point $(-1, -2)$.

(c) Evaluate $\frac{d^2y}{dx^2}$ at the point $(-1, -2)$.

(d) The point $(\frac{1}{6}, 0)$ is on the curve. Find the value of $(y^{-1})'(0)$.

~~也同时对x求导~~
(a) $3(2y+1)^2 \cdot 2 \cdot \frac{dy}{dx} - 24 = 0$.

Chain Rule. $6(2y+1)^2 \frac{dy}{dx} = 24$

$$\frac{dy}{dx} = \boxed{\frac{4}{(2y+1)^2}}$$

At $(-1, -2)$

(b) $\frac{dy}{dx} = \frac{4}{(-4+1)^2} = \boxed{\frac{4}{9}}$

$$\boxed{y+2 = \frac{4}{9}(x+1)}$$

(d) $(y^{-1})'(y) = \frac{1}{y'(x)}$

Since $(\frac{1}{6}, 0)$,
 $y(\frac{1}{6}) = 0$.

$$(y^{-1})'(0) = \frac{1}{y'(\frac{1}{6})}$$

$$y' = \frac{4}{(2y+1)^2} = \frac{4}{1} = 4$$

$$(y^{-1})'(0) = \frac{1}{4}$$

$$= 4(2y+1)^{-2}$$

$\frac{dy}{dx} = \frac{4}{(2y+1)^2}$ 也再次同时 对x求导

$$\frac{d^2y}{dx^2} = 4 \cdot (-2)(2y+1)^{-3} \cdot 2 \cdot \frac{dy}{dx}$$

$$= -16(2y+1)^{-3} \cdot \frac{dy}{dx}$$

at the point $(-1, -2)$

$$\frac{dy}{dx} = \frac{4}{9} \text{ substitute in.}$$

$$\frac{d^2y}{dx^2} = -16(-4+1)^{-3} \times \frac{4}{9}$$

$$= -16 \times (-3)^{-3} \times 4 \times 3^{-2}$$

$$= \frac{-64}{-3^5} = \boxed{\frac{64}{243}}$$

inverse function

求导

下来!!

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x	-3	-2	-1	1
$f(x)$	$-\frac{5}{2}$	-3	-2	$\frac{2}{3}$
$f'(x)$	-1	$\frac{1}{3}$	$\frac{6}{5}$	$\frac{4}{3}$

当 $f(x)$ 中的 x 不是 single x ,
求导的时候, 一定要 chain Rule!

The table above gives values of the differentiable function f and its derivative for selected values of x .

(a) Let g be the function defined by $g(x) = \frac{f(x^2)}{e^x}$. Find $g'(-1)$.

(b) Let h be the function defined by $h(x) = f(f(-2x))$. Find $h'(1)$.

(c) Let k be the function defined by $k(x) = f(x) \cdot \arcsin\left(\frac{x}{2}\right)$. Find $k'(-1)$.

$$(a) g'(x) = \frac{f'(x^2) \cdot 2x \cdot e^x - e^x f(x^2)}{(e^x)^2}$$

$$g'(-1) = \frac{\frac{4}{3} \times (-2) \times \frac{1}{e} - \frac{1}{e} \times \frac{2}{3}}{\left(\frac{1}{e}\right)^2}$$

$$= \frac{\frac{1}{e} \left(-\frac{8}{3} - \frac{2}{3}\right)}{\left(\frac{1}{e}\right)^2}$$

$$= \frac{-\frac{10}{3}}{\frac{1}{e}} = \boxed{-\frac{10}{3}e}$$

$$(b) h'(x) = f'(f(-2x)) \cdot f'(-2x) \cdot (-2x)$$

$$= f'(f(-2x)) \cdot f'(-2x) \cdot (-2)$$

$$h'(1) = f'(f(-2)) \cdot f'(-2) \cdot (-2)$$

$$= f'(-3) \cdot \frac{1}{3} \times (-2)$$

$$= -1 \times \frac{1}{3} \times (-2) = \boxed{\frac{2}{3}}$$

$$(c) k'(x) = f'(x) \cdot \arcsin\left(\frac{x}{2}\right) + \left[\arcsin\left(\frac{x}{2}\right)\right]' \cdot f(x)$$

$$= f'(x) \arcsin\left(\frac{x}{2}\right) + \frac{1}{\sqrt{1 - \left(\frac{x}{2}\right)^2}} \times \frac{1}{2} \cdot f(x)$$

$$k'(-1) = f'(-1) \arcsin\left(-\frac{1}{2}\right) + \frac{1}{2\sqrt{1 - \left(-\frac{1}{2}\right)^2}} \times f(-1)$$

$$= \frac{6}{5} \times \left(-\frac{\pi}{6}\right) + \frac{1}{\sqrt{3}} \times (-2) = \boxed{-\frac{\pi}{5} - \frac{2}{\sqrt{3}}}$$

chain Rule到底!

剥洋葱般,
一层一层

$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$
剥洋葱般,
 $(\arccos x)'$ 类似
 $(\arctan x)'$ 类似